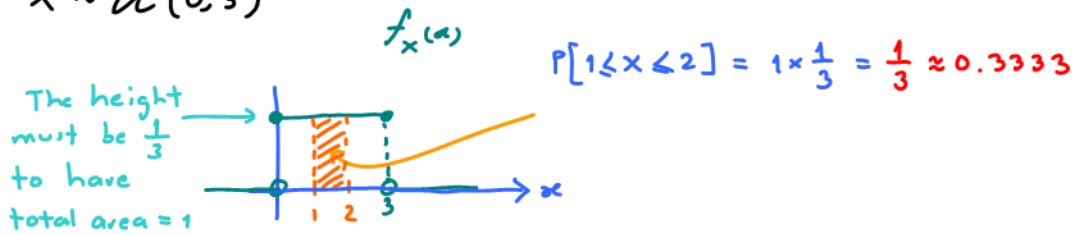


ECS315 2014 Quiz 5 Solution

Evaluate $P[1 < X < 2]$ for each of the following RVs

(a) $X \sim \mathcal{U}(0, 3)$



Alternatively, the cdf of $\mathcal{U}(0, 3)$ is

$$F_x(x) = \begin{cases} \frac{x-0}{3-0}, & 0 \leq x \leq 3, \\ 0, & \text{otherwise} \end{cases}$$

Therefore, $P[1 \leq X \leq 2] = F_x(2) - F_x(1) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

(b) $X \sim \mathcal{E}(3)$

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$$f_x(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases} = \begin{cases} 3e^{-3x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$P[1 \leq X \leq 2] = \int_1^2 f_x(x) dx = \int_1^2 3e^{-3x} dx = \left. \frac{3e^{-3x}}{-3} \right|_1^2 = e^{-3} - e^{-6} \approx 0.0473$$

Alternatively, the cdf of $\mathcal{E}(3)$ is

$$F_x(x) = \begin{cases} 1 - e^{-3x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Therefore, $P[1 \leq X \leq 2] = F_x(2) - F_x(1) = (1 - e^{-6}) - (1 - e^{-3}) = e^{-3} - e^{-6}$

(c) $X \sim \mathcal{N}(0, 1)$

$$P[1 \leq X \leq 2] = \Phi(2) - \Phi(1) \approx 0.97725 - 0.8413 \approx 0.1359$$

(d) $X \sim \mathcal{N}(1, 3)$

$$P[1 \leq X \leq 2] = \Phi\left(\frac{2-1}{\sqrt{3}}\right) - \Phi\left(\frac{1-1}{\sqrt{3}}\right) = \Phi\left(\frac{1}{\sqrt{3}}\right) - \Phi(0) \approx \Phi(0.58) - 0.5$$

$$\approx 0.7190 - 0.5 = 0.2190$$